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An Approximate Formula for Calculating Z_0 of a Symmetric Strip Line

The equations available to determine the characteristic impedance of a symmetric strip line (Fig. 1) to high accuracy are difficult to utilize without a computer [1],[2]. Other less accurate equations which are easier to apply have been developed, and the one most frequently quoted has been given by Cohn [3]:

$$Z_0 = \frac{94.15/\sqrt{\epsilon_r}}{[W/(D-T) + C_0/\pi]} \qquad (1)$$

where

$$C_0 = \frac{2}{1-T/D}\ln\left[\frac{1}{1-T/D}+1\right] - \left[\frac{1}{1-T/D}-1\right] \cdot \ln\left[\frac{1}{(1-T/D)^2}-1\right]. \qquad (2)$$

Equation (1) was stated by Cohn to be applicable over the range $W/(D-T)\geq 0.35$ and $T/D\leq 0.25$, with a maximum error of approximately 1 percent at the lower limit of W/D. Good agreement between computed and measured values of Z_0 has also been obtained for values of W/D and T/D outside Cohn's stated limits [4].

Chen [5] has supplied another equation to determine Z_0 for a symmetric strip line:

$$Z_0 = \frac{94.15/\sqrt{\epsilon_r}}{[W/(D-T) + C_e/\pi} \qquad (3)$$

where

$$C_e = \frac{D}{D-T}\ln\left[\frac{2D-T}{T}\right] + \ln\left[\frac{T(2D-T)}{(D-T)^2}\right]. \qquad (4)$$

Although not immediately obvious, and apparently not realized by some [4], (2) and (4) are identical. By substituting $x=1/(1-T/D)$,

Manuscript received July 8, 1966.

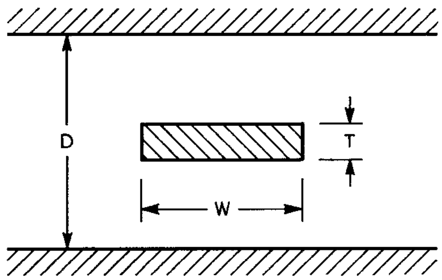

Fig. 1. Cross section of symmetric slab line.

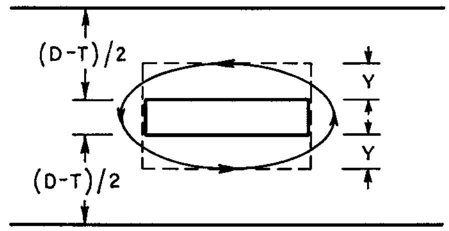

Fig. 2. Approximation to line of magnetic field intensity.

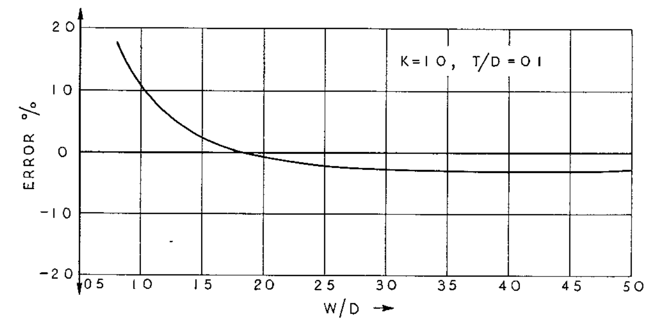

Fig. 3. Percentage error in characteristic impedance.

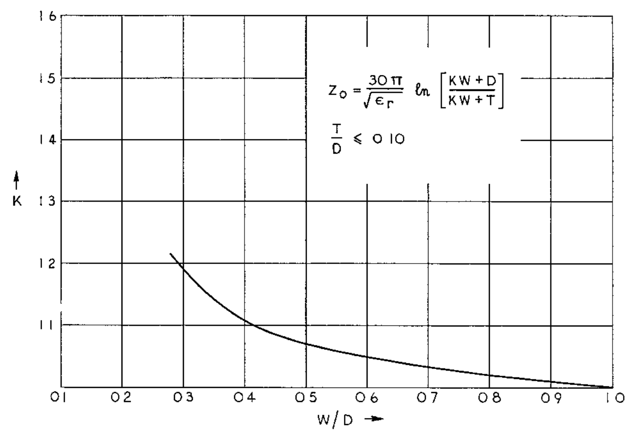

Fig. 4. Variation of correction factor with W/D ratio.

$$C_0 = 2x\ln(x+1) - (x-1)\ln(x^2-1) = \ln\left[\frac{(x+1)^{x+1}}{(x-1)^{x-1}}\right].$$

From (4),

$$C_e = \frac{1}{1-T/D}\ln\left[\frac{2-T/D}{T/D}\right] + \ln\left[\frac{(T/D)(2-T/D)}{(1-T/D)^2}\right].$$

Now $T/D=1-1/x$ and $2-T/D=1+1/x$.

$$\therefore\ C_e = x\ln\left[\frac{1+1/x}{1-1/x}\right] + \ln\left[(1-1/x)(1+1/x)x^2\right] = \ln\left[\frac{(x+1)^{x+1}}{(x-1)^{x-1}}\right].$$

Hence, $C_0=C_e$ and (1)–(4) can be combined together in the following expression:

$$Z_0 = \frac{94.15}{\sqrt{\epsilon_r}\left[\frac{W}{D}\cdot x + \frac{1}{\pi}\ln\left\{\frac{(x+1)^{x+1}}{(x-1)^{x-1}}\right\}\right]}. \qquad (5)$$

However, (5) is not a particularly easy one to handle, and a simpler although less comprehensive one has been developed, accurate to better than 1.2 percent of (5) for $W/D\geq 1.0$, and to within 5 percent for $W/D\geq 0.75$, providing that in both cases $T/D\leq 0.2$. It is

$$Z_0 = \frac{30\pi}{\sqrt{\epsilon_r}}\ln\left(\frac{W+D}{W+T}\right). \qquad (6)$$

Equation (6) can be obtained by considering the "average" length of the lines of magnetic field intensity surrounding the central conductor of a strip line. The characteristic impedance of a uniform transmission line operating in the transverse electromagnetic (TEM) mode can be determined by first calculating the inductance per unit length of the line L then applying the equation

$$Z_0 = c_0'L \qquad (7)$$

where c_0' is the velocity of a TEM wave in an infinite medium of dielectric, relative permittivity ϵ_r.

The inductance per unit length is given by

$$L = \frac{\Phi}{lI} \tag{8}$$

where

$$\Phi = \int_A B \cdot dA \quad \text{(Gaussian Law)} \tag{9}$$

and

$$I = \oint H \cdot ds \quad \text{(Ampere's Law)}. \tag{10}$$

Consider now a closed loop of a typical line of magnetic field around the center conductor of a slab line (Fig. 2). If such a loop is approximated by a rectangular one of about the same length, $2W+2T+4y$, the magnetic field intensity H at distance y from the center conductor can be calculated from (10) and is found to be

$$H = \frac{I}{2W + 2T + 4y}. \tag{11}$$

Substituting for the magnetic flux density, $B(=\mu H)$ in (9), changing the variable of integration from A to y with $dA = ldy$, putting in appropriate limits of integration and rearranging terms,

$$\frac{\Phi}{lI} = L = \int_0^{(D-T)/2} \frac{\mu dy}{2W + 2T + 4y} \tag{12}$$

from which it readily follows that

$$L = \frac{\mu}{4} \ln \left[\frac{W + D}{W + T}\right]. \tag{13}$$

Substituting for L in (7), putting in the values for c_0' and μ, (6) is obtained.

A graphical comparison of (5) and (6) is shown in Fig. 3 where the fractional error in characteristic impedance is plotted as a function of W/D for $T/D = 0.1$. For $W/D < 1.0$ the error in calculating Z_0 using (6) becomes increasingly greater than 1 percent. However, its range of application can be extended by considering a longer line of magnetic field, of length $2KW+2T+4y$, where $K > 1$ and is a function of W/D. Equation (6) modifies to

$$Z_0 = \frac{30\pi}{\sqrt{\epsilon_r}} \ln \left(\frac{KW + D}{KW + T}\right). \tag{14}$$

By using (5) and (14), values of K as a function of W/D have been found and plotted graphically (Fig. 4), which allow (14) to be used to determine Z_0 to within 0.7 percent of Cohn's value, for $W/D \geq 0.30$, and $T/D \leq 0.10$. For $T/D \leq 0.20$, Z_0 can be determined to within about 5 percent using (14) for $W/D \geq 0.30$.

Finally, for the special case of zero strip thickness ($T/D = 0$), (5) simplifies to the well-known expression

$$Z_0 = \frac{94.15/\sqrt{\epsilon_r}}{\left[\frac{W}{D} + \frac{\ln 4}{\pi}\right]}. \tag{15}$$

Similarly, (6) reduces to

$$Z_0 = \frac{30\pi}{\sqrt{\epsilon_r}} \ln \left[1 + \frac{D}{W}\right] \tag{16}$$

which is accurate to within almost 1 percent of (15) for $W/D \geq 0.75$.

ACKNOWLEDGMENT

The permission of the Postmaster-General's Department, Australia, to publish this letter is gratefully acknowledged.

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An Improved Method for Measuring Scattering Parameters of Nonreciprocal Two-Ports

Usual methods for measuring the characteristic parameters of reciprocal structures fail when applied to nonreciprocal structures. Methods which rest on the use of the input impedance formula

$$Z_{in} = \frac{(Z_{11}Z_{22} - Z_{12}Z_{21}) + Z_{11}Z_{out}}{Z_{22} + Z_{out}}$$

or, of the input reflection coefficient relationship

$$K_{in} = S_{11} - \frac{S_{12}S_{21}}{S_{22} - \frac{1}{K_{out}}}$$

allow determination of the products $Z_{12}Z_{21}$ or $S_{12}S_{21}$, but are unable to separate the individual factors. Another difficulty arises when one considers that practical magnitudes of the scattering parameters often lie very close to zero or unity (e.g., for isolators or for phase shifters). The method which we propose allows accurate determination of both magnitude and phase of the scattering parameters. The procedure gives good results for arbitrary values of the parameters, and is an extension of a procedure suggested by Macpherson [1] and Pippin [2], and adapted by Altschuler [3].

Consider a nonreciprocal two-port with reference planes 1 and 2 (Fig. 1). Then,

$$b_1 = S_{11}a_1 + S_{12}a_2 \tag{1}$$

$$b_2 = S_{21}a_1 + S_{22}a_2. \tag{2}$$

Let us assume that $|a_1| = |a_2|$ and $a_1/a_2 = ej\theta$. The reflection coefficient at the reference

Manuscript received July 21, 1966; revised September 26, 1966.

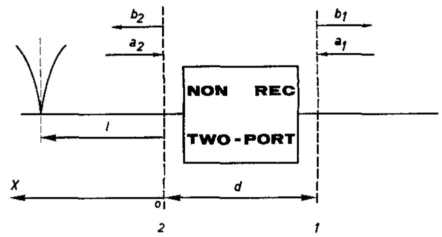

Fig. 1. Nonreciprocal two-port with its reference indications.

plane 2, K_2, then takes the value, from (2),

$$K_2 = \frac{b_2}{a_2} = S_{22} + S_{21}e^{j\theta}. \tag{3}$$

By means of a calibrated phase shifter at one side of the two-port, θ can be adjusted to take the values 0 or π. This yields

$$(K_2)_0 = S_{22} + S_{21} \tag{4}$$

$$(K_2)_\pi = S_{22} - S_{21}. \quad \text{(Pippin)} \tag{5}$$

From (4) and (5) S_{22} and S_{21} can be readily computed, but with the poor accuracy attached to a one point method. Altschuler improved the situation by remarking that, when θ varies from 0 to 2 π, K_2 describes a circle in the complex plane. The center of the circle corresponds to S_{22} and the radius to $|S_{21}|$. The use of a large number of points now allows averaging of the experimental errors. With practical structures, however, this method still is not satisfactory. If S_{22} and S_{21} differ too much, for example, it is impossible to locate the center of the circle with accuracy. Furthermore, high VSWR must be measured, which is both difficult and inaccurate. If S_{22} and S_{21} are both very close to zero, very low VSWR must be measured, and the precision drops again because of the mismatch errors. Accurate phase determination is still difficult because it depends on a one point measurement. To improve the method, let us consider the equation

$$K_2 = S_{22} + S_{21}\frac{a_1}{a_2}, \tag{6}$$

which can be rewritten as

$$K_2 = S_{22} + S_{21}\left|\frac{a_1}{a_2}\right| e^{j(\arg a_1 - \arg a_2)}. \tag{7}$$

The radius of the circle is now $|S_{21}| \cdot |a_1/a_2|$. If we succeed in adjusting $|a_1/a_2|$ to any desired value, the original circle of Altschuler will be compressed or expanded and the measurements become both easy and accurate. Besides, really accurate measurements can be performed by utilizing several values of $|a_1/a_2|$, i.e., by plotting several circles in the complex plane. From each circle, a value $|S_{21}|$ can be obtained and the various values can be averaged. This procedure smooths out the errors of the attenuators. Points with constant phase θ must lie on straight lines through the common center of the circles. Phase determination, therefore, becomes accurate too (Fig. 2). The practical set-up is shown in Fig. 3. The matched pads consist of an isolator matched with an E-H tuner to avoid multiple reflections. The measurements proceed as follows:

1) The structure under test is replaced by a piece of waveguide with the same